NONLINEAR SURFACE ACOUSTIC WAVES IN CUBIC CRYSTALS

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OUTLINE

• Background
  – Nonlinear surface waves
  – Surface waves in crystals
  – Nonlinear theory

• Results in (001) plane
  – Simulations
  – Experiment

• Results in (111) plane
  – Simulations
  – Experiment
  – Approximate Method

• Summary
NONLINEAR SURFACE WAVES

Schematic Diagram:

Waveform Distortion:
(in isotropic media or certain mirror planes of cubic crystals)
SURFACE WAVES IN CRYSTALS

Schematic Diagram:

Typical Planes for Crystal Cuts:

(001) plane

(111) plane
NONLINEAR THEORY

Approach: Hamiltonian mechanics formalism
(Hamilton, Il’inskii, Zabolotskaya, 1996)

Velocity waveforms in solid:
\[ v_j(x, z, t) = \sum_{n=-\infty}^{\infty} v_n(x) u_{nj}(z) e^{in(kx-\omega t)} \]
\[ u_{nj}(z) = \sum_{s=1}^{3} \beta_j^{(s)} e^{ink_3^{(s)} z} \]

Coupled spectral evolution equations:
\[ \frac{dv_n}{dx} + \alpha_n v_n = -\frac{n^2 \omega c_{44}^4}{2 \rho c^4} \sum_{l+m=n} \frac{lm}{|lm|} \hat{S}_{lm} v_l v_m \]

- \( v_n \rightarrow n\text{th harmonic amplitude} \)
- \( \hat{S}_{lm} \rightarrow \text{nonlinearity matrix elements} \)
- \( \alpha_n \rightarrow \text{weak attenuation} \)

Observation:
(001) plane \( \Rightarrow \) \( S_{lm} \) real valued
(111) plane \( \Rightarrow \) \( S_{lm} \) complex valued
NONLINEARITY MATRIX FOR SI (001)

Nonlinearity matrix elements:

\[
\hat{S}_{lm} = \frac{-1}{c_{44}} \sum_{s_1,s_2,s_3=1}^{3} \frac{1}{2} d'_{ijpqrs} \beta_i^{(s_1)} \beta_p^{(s_2)} [\beta_r^{(s_3)}]^* \lambda_j^{(s_1)} \lambda_q^{(s_2)} [\lambda_s^{(s_3)}]^*}{l \lambda_3^{(s_1)} + m \lambda_3^{(s_2)} - (l + m) [\lambda_3^{(s_3)}]^*}
\]

\(\lambda_i^{(s)} \rightarrow\) eigenvalues of linear problem
\(\beta_j^{(s)} \rightarrow\) eigenvectors of linear problem
\(d'_{ijpqrs} \rightarrow\) 2nd and 3rd order elastic constants

Selected matrix elements for Si in (001) plane:

![Graph showing selected matrix elements for Si in (001) plane]
SIMULATIONS WITH SINUSOIDS FOR SI (001)

V_x

\( X = 0 \)
\( X = 1 \)
\( X = 2 \)

V_z

\( X = 0 \)
\( X = 1 \)
\( X = 2 \)

Si in (001) plane
EXPERIMENT

Approach: Laser-excited photoelastic SAW generation (Lomonosov and Hess, 1996)

- Pulse detection:
  Probe beam deflection proportional to vertical vel.
  Temporal resolution: 1 ns

- Beam locations:
  1st probe beam: 5 mm from source
  2nd probe beam: 10–15 mm from 1st probe beam

- Resulting SAW pulses:
  Duration: 20 to 40 ns
  Peak strain: 0.005 to 0.010 (near fracture)
EXPERIMENT FOR SI (001)

Longitudinal velocity waveforms at close location:

Longitudinal velocity waveforms at remote location:
NONLINEARITY MATRICES IN (001) PLANE

Other crystals investigated:
RbCl, BaF₂, CaF₂, SrF₂, Al, Ni, Cu, C (diamond)
OTHER EFFECTS IN (001) PLANE

Si (001), $\theta \approx 21^\circ$: $\hat{S}_{lm} \approx 0$

KCl (001), $\theta \approx 3^\circ$: $\hat{S}_{11} > 0$, $\hat{S}_{12} \approx 0$, $\hat{S}_{13} < 0$

KCl (001), $\theta \approx 10^\circ$: $|\hat{S}_{11}| < |\hat{S}_{12}| < |\hat{S}_{13}|$
NONLINEARITY MATRICES IN (111) PLANE

\[ |S_{lm}| \]

\[ \psi_{lm} \text{ [rad]} \]

\[ \theta \text{ [deg]} \]

\[ \eta = 0.803 \]

\[ \eta = 1.02 \]

\[ \eta = 2.60 \]

\[ \eta = 3.20 \]
SIMULATIONS WITH SINUSOIDS FOR SI (111)

Velocity waveform in direction 0° from ⟨112⟩:

Nonlinearity matrix elements:
EXPERIMENT FOR SI (111)

Propagation in direction $0^\circ$ from $\langle 11\bar{2} \rangle$

Velocity waveforms at close location:

Velocity waveforms at remote location:
Define the matrix element transformation
\[ \hat{S}_{lm}^\psi = \hat{S}_{lm} e^{i \text{sgn}(n) \psi}, \quad n = l + m. \]

The evolution equations with the transformed matrix are
\[
\frac{dv_n^\psi}{dx} = -\frac{n^2 \omega_0 c_{44}}{2 \rho c^4} \sum_{l+m=n} \frac{lm}{|lm|} \hat{S}_{lm} v_l^\psi v_m^\psi
\]
\[
= -\frac{n^2 \omega_0 c_{44}}{2 \rho c^4} \sum_{l+m=n} \frac{lm}{|lm|} \hat{S}_{lm} e^{i \text{sgn}(n) \psi} v_l^\psi v_m^\psi
\]

If the transformed spectral amplitudes are
\[ v_n^\psi = v_n e^{-i \text{sgn}(n) \psi}, \]
then it follows that
\[
\frac{dv_n}{dx} = -\frac{n^2 \omega_0 c_{44}}{2 \rho c^4} \sum_{l+m=n} \frac{lm}{|lm|} \hat{S}_{lm} v_l v_m.
\]

**Result:** Equations relate phase \( \psi \) of matrix elements to spectral amplitudes (and thus waveforms).
PHASE TRANSFORMED WAVEFORMS

Consider artificial example of Rayleigh waves in steel

\[ \hat{S}_{lm} \rightarrow \text{positive, real valued matrix elements} \]
\[ \psi_n \rightarrow \text{corresponding spectral components} \]

under the transformation \( \psi_n^\psi = \psi_n e^{-i(\text{sgn } n)} \).

Resulting longitudinal velocity waveforms:
APPROXIMATE AND FULL SOLUTIONS

**Approximation:**

Steel with $\psi = 0.59\pi$

Steel with $\psi = 0.39\pi$

**Full Solution:**

Si (111) 0°

KCl (111) 20°
SUMMARY

Results:

• Investigations of nonlinear SAWs in cubic crystals for:
  – Different materials, cuts, directions

• Nonlinearity matrix is useful for characterizing waveform distortion.

• In (001) plane:
  – Sensitive dependence of nonlinearity on direction
  – Compression and rarefaction shocks in same plane
  – Shock suppression in certain directions
  – Agreement between measured and calculated waveforms

• In (111) plane:
  – Asymmetric waveform distortion
  – Low frequency oscillations due to phase differences
  – Agreement between measured and calculated waveforms
  – Development of approximate method which works well where phases between elements are similar