NONLINEAR SURFACE ACOUSTIC WAVES IN CUBIC CRYSTALS

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Ultrasonics International 2001
Delft, The Netherlands
SUMMARY OF RESULTS

• A nonlinearity matrix that describes the coupling between harmonics is shown to provide a useful tool for characterizing waveform distortion in different crystals, surface cuts, and propagation directions.

• In the (001) surface cut:
  – Distortion type depends sensitively on direction.
  – Example: longitudinal velocity component varies between compression and rarefaction shocks.
  – Calculated waveforms corroborated by measured data in crystalline silicon.

• In the (111) surface cut:
  – Initially sinusoidal waveforms exhibit asymmetric distortion as they propagate.
  – Harmonic phase plays an important role in the distortion process.
  – Calculated waveforms are corroborated by measured data in crystalline silicon.
NONLINEAR SURFACE WAVES

Surface waves are good candidates for shock formation because:

- Energy is concentrated near the surface.
- All harmonics travel at the same wave speed (i.e., nondispersive).

Waveform distortion in isotropic media:

- Waveforms are “snapshots” in frame moving at wave speed.
- Propagation distance X scaled by shock formation distance.
Velocity waveforms in solid:

\[ v_j(x, z, t) = \sum_{n=-\infty}^{\infty} v_n(x) u_{nj}(z) e^{in(kx-\omega t)} \]

\[ v_n(x) \rightarrow n\text{th harmonic amplitude} \]
\[ u_{nj}(z) \rightarrow \text{depth functions from linear solution} \]

Coupled spectral evolution equations:

\[ \frac{dv_n}{dx} + \alpha_n v_n = -\frac{n^2 \omega}{2 \rho c^4} \sum_{l+m=n} \frac{lm}{|lm|} \hat{S}_{lm} v_l v_m \]

The nonlinearity matrix elements \( \hat{S}_{lm} \) are:

- coefficients which represent coupling of \( l \)th and \( m \)th harmonics to generate \( n = (l + m) \)th harmonic,
- only a function of density, 2nd and 3rd order elastic constants, and the solution of the linear problem,
- useful for mapping the types of waveform distortion as a function of propagation direction.
REAL-VALUED NONLINEARITY: (001) CUT

- Plane of mirror symmetry.
- All $\hat{S}_{lm}$ are real-valued.

Angular dependence of $\hat{S}_{lm}$ in Si:

- Surface waves exhibit 3 distinct regions of nonlinearity with corresponding variation in waveform distortion.
Top waveforms show positive nonlinearity (compression shock in $V_x$) in Region II.

Bottom waveforms show negative nonlinearity (rarefaction shock in $V_x$) in Region I.
EXPERIMENT: SI (001), 0° and 26° from ⟨100⟩

Method: Laser surface wave generation and detection
[from A. Lomonosov and P. Hess, University of Heidelberg]

Longitudinal velocity component, 5 mm from source:

Same pulses, 20 mm from source:

- Rarefaction shock forms at $\theta = 0°$, while compression shock forms at $\theta = 26°$.
- Waveforms are similar at $x=5$ mm, but quite different at $x=20$ mm.
**NONLINEARITY MATRICES: (001) PLANE**

- Note diversity of $\hat{S}_{lm}$ for a variety of crystals.
- Plots are ordered by the anisotropy ratio $\eta = c_{44}/(c_{11} - c_{12})$. For isotropic media, $\eta = 1$. 

![Graphs showing nonlinearities for various crystals](image-url)
Angular dependence of $\hat{S}_{lm}$ in Si:

- $\hat{S}_{lm}$ are complex-valued in most directions.

- $\arg(\hat{S}_{lm}) = 0$ corresponds to positive nonlinearity.

- $\arg(\hat{S}_{lm}) = \pi$ corresponds to negative nonlinearity.
SIMULATIONS WITH SINUSOIDS: SI (111)

- Top waveforms show complex-valued $\hat{S}_{lm}$ at $\theta = 0^\circ$ result in asymmetric distortion.
- Bottom waveforms show negative, real-valued $\hat{S}_{lm}$ at $\theta = 30^\circ$ result in symmetric distortion.
EXPERIMENT: SI (111), 0° from ⟨112⟩

Method: Laser surface wave generation and detection [from A. Lomonosov and P. Hess, University of Heidelberg]

Velocity waveforms, 5 mm from source:

Same pulses, 21 mm from source:

- Waveforms distort similarly to the sinusoids ($v_x$ distorts in U-shape; $v_z$, in N-shape).
- Note $v_x$ is differs from (001) cut even with the same excitation process.
\( \hat{S}_{lm} \) for a variety of crystals, ordered by anisotropic ratio.

Although the magnitudes show similar trends, the phases vary from case to case \( \psi_{lm} = \arg(\hat{S}_{lm}) \).