A SIMPLE METHOD FOR COMPARING WAVEFORM DISTORTION OF NONLINEAR SURFACE WAVES IN DIFFERENT CUBIC CRYSTALS

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OUTLINE

• Nonlinear Surface Waves

• Nonlinear Theory

• Comparison with Experiment

• Diversity of Nonlinear Distortion in Crystals

• Simple Approximate Method

• Comparison of Full and Approximate Methods
NONLINEAR SURFACE WAVES

Schematic Diagram:

Waveform Distortion:
(in isotropic media or certain mirror planes of cubic crystals)
NONLINEAR THEORY

Approach: Hamiltonian mechanics formalism
(Hamilton, Il’inskii, Zabolotskaya, 1996)

Velocity waveforms in solid:

\[ v_j(x, z, t) = \sum_{n=-\infty}^{\infty} v_n(x) u_{nj}(z) e^{in(kx-\omega t)} \]

\[ u_{nj}(z) = \sum_{s=1}^{3} q_j^{(s)} e^{inkl_j^{(s)} z} \]

Coupled spectral evolution equations:

\[ \frac{dv_n}{dx} + \alpha_n v_n = \frac{n^2 \omega}{2 \rho c^4} \sum_{l+m=n} \frac{lm}{|lm|} S_{lm} v_l v_m \]

- \( v_n \rightarrow \) \( n \)th harmonic amplitude
- \( S_{lm} \rightarrow \) nonlinearity matrix elements
- \( \alpha_n \rightarrow \) weak attenuation

Observation:

Higher symmetry cases \( \Rightarrow \) \( S_{lm} \) real valued
Lower symmetry cases \( \Rightarrow \) \( S_{lm} \) complex valued
CRYSTALLINE GEOMETRY

Typical Planes for Crystal Cuts:

(001) plane

(111) plane
COMPARISON WITH EXPERIMENT
[data from A. Lomonosov and P. Hess (1998, 1999)]

Si, (001) plane, $26^\circ$ from $\langle 100 \rangle$  Si, (111) plane, $0^\circ$ from $\langle 11\bar{2} \rangle$

Velocity waveforms at close location:

Velocity waveforms at remote location:

- $v_x$ vs. $t$ graphs for the close location.
- $v_x$ vs. $t$ graphs for the remote location.

- Theory vs. Experiment comparison in the graphs.
WAVEFORM DISTORTION IN (111) PLANE

(111)\text{KCl}\quad<112>

\text{Longitudinal Velocity Waveforms}

\text{Vertical Velocity Waveforms}

\text{Si}

\text{Longitudinal Velocity Waveforms}

\text{Vertical Velocity Waveforms}

(111)\text{KCl}\quad X=2
NONLINEARITY MATRIX

Nonlinearity matrix is:

- Only a function of density, 2nd and 3rd order elastic constants, and the eigenvalues and eigenvectors of the linear problem,
- Simpler to compute than a full integration of model eqs.

Si (111)  

| \(|S_{lm}|\) |
|---|
| 0.12 |
| 0.1 |
| 0.08 |
| 0.06 |
| 0.04 |
| 0.02 |
| 0 |

0° 10° 20° 30°  

Angle from \(\langle 112 \rangle\)

<table>
<thead>
<tr>
<th>(\arg(S_{lm}))</th>
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</thead>
<tbody>
<tr>
<td>180°</td>
</tr>
<tr>
<td>90°</td>
</tr>
<tr>
<td>0°</td>
</tr>
<tr>
<td>-90°</td>
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<tr>
<td>-180°</td>
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0° 10° 20° 30°  

Angle from \(\langle 112 \rangle\)

Query:
How is the phase of the matrix elements related to the shape of waveform distortion?
PHASE OF MATRIX ELEMENTS

Define the transformation

\[ S_{lm}^\theta = S_{lm} e^{i(\text{sgn } n)\theta}, \quad n = l + m. \]  

(1)

The evolution equations with the transformed matrix are

\[
\frac{dv_n^\theta}{dx} = \frac{n^2 \omega_0}{2\rho c^4} \sum_{l+m=n} \frac{lm}{|lm|} S_{lm}^\theta v_l^\theta v_m^\theta.
\]

Defining the relation

\[ v_n^\theta = v_n e^{-i(\text{sgn } n)\theta} \]

(2)

implies that the evolution equations can be rewritten as

\[
\frac{dv_n}{dx} = \frac{n^2 \omega_0}{2\rho c^4} \sum_{l+m=n} \frac{lm}{|lm|} S_{lm} v_l v_m.
\]

Result:

Eqs. (1) and (2) show how the phase of the matrix elements are related to the phase of the spectral components (and therefore the waveform shape).
PHASE TRANSFORMED WAVEFORMS

Consider Rayleigh waves in steel

\[ S_{lm} \rightarrow \text{positive, real valued matrix elements} \]

\[ v_n \rightarrow \text{corresponding spectral components} \]

under the transformation \( v_n^\theta = v_n e^{-i \text{sgn} n \theta} \).

Resulting longitudinal velocity waveforms:
ELEMENTS WITH SIMILAR PHASE

Si (111) 0°

KCl (111) 0°

Steel with θ=105.9°

Steel with θ=-149.7°

Si (111) 0°

KCl (111) 0°
ELEMENETS WITH DISSIMILAR PHASE

KCl (111) 20°

KCl (111) 28°

Steel with θ=70.9°

Steel with θ=85.0°

KCl (111) 20°

KCl (111) 28°
CONCLUSION

Results:

• The phase of the nonlinearity matrix elements was shown to be key to describing waveform distortion in cubic crystals.

• An approximate method for characterizing waveform evolution using this phase was developed.

• For similarly phased matrix elements, the dominant matrix element provided a good approximation of waveform distortion.

• For dissimilarly phased matrix elements, the full solution of the model equations must be performed.