PULSED NONLINEAR SURFACE ACOUSTIC WAVES IN CRYSTALS

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• Anisotropy in Crystalline Silicon

• Theory

• Experiment

• Diffraction Effects

• Simulations with Sinusoids

• Comparison of Experiment and Theory

• Conclusion & Future Work
ANISOTROPY IN CRYSTALLINE SILICON

Stress-strain relation for cubic crystal:

\[ \sigma_{ij} = c_{ijkl}e_{kl} + d_{ijklmn}e_{kl}e_{mn} \]

- \( c_{ijkl} \rightarrow 3 \) Second Order Elastic (SOE) constants
- \( d_{ijklmn} \rightarrow 6 \) Third Order Elastic (TOE) constants

Data for Si elastic constants:


Diamond Cubic Structure: Crystal Cut in Experiment:
Schematic Diagram of a Surface Acoustic Wave in an Anisotropic Medium
THEORY

Approach: Hamiltonian mechanics formalism
(Hamilton, Il’inskii, Zabolotskaya, 1996)

Velocity waveforms in solid:
\[ v_j(x, z, t) = \sum_{n=-\infty}^{\infty} v_n(x) u_{nj}(z) e^{in(kx-\omega t)} \]

Coupled spectral evolution equations:
\[ \frac{dv_n}{dx} + \alpha_n v_n = -n^2 \sum_{l+m=n} \frac{l m}{|lm|} R_{lm} v_l v_m \]

Solve equations numerically:
- Input data
  - Material constants (density, SOE, TOE)
  - Waveform spectrum \((x = 5 \text{ mm})\)
- Apply 4th order Runge-Kutta routine with:
  - Number of harmonics: 400
  - Pulse repetition frequency: 10 MHz
  - Maximum bandwidth: 4000 MHz

  Weak absorption was added for numerical stability.
EXPERIMENT

Approach: Laser-excited thermoelastic SAW generation
(Lomonosov and Hess, 1996)

Pulse detection:
Probe beam deflection proportional to vertical vel.
Photodiode bandwidth: 500 MHz

Beam locations:
Laser excitation: $x = 0$ mm
1st probe beam: $x = 5$ mm
2nd probe beam: $x = 21$ mm

Resulting SAW pulses:
Duration: 20 to 40 ns
Peak strain: 0.005 to 0.010 (near fracture)
Measured frequency spectrum at $x = 5$ mm:

![Graph showing measured frequency spectrum with characteristic frequency at 50 MHz.]

Characteristic frequency: 50 MHz

- **Analysis:**
  - Characteristic beam radius: $a = 3$ mm
  - Diffraction length for SAW beam: $\frac{1}{2}ka^2 = 300$ mm
  - Furthest measurement distance: $x = 21$ mm

- **Conclusion:**
  - Diffraction effects are not important.
SIMULATIONS WITH SINUSOIDS

Calculated vertical velocity waveforms ($f_0 = 50$ MHz):

\begin{figure}
\centering
\includegraphics[width=\textwidth]{vertical_velocity_waveform}
\end{figure}

Calculated longitudinal velocity waveforms:

\begin{figure}
\centering
\includegraphics[width=\textwidth]{longitudinal_velocity_waveform}
\end{figure}
EVOLUTION OF WAVEFORMS

Velocity waveforms at $x = 5$ mm:

Velocity waveforms at $x = 21$ mm:
EFFECT OF RECONSTRUCTION BANDWIDTH

Consider the longitudinal velocity waveforms at $x = 21$ mm. Theoretical waveforms with shaded bandwidth of 700 MHz:

![Theoretical waveforms with bandwidth of 700 MHz](image)

Theoretical waveforms with bandwidth of 3000 MHz:

![Theoretical waveforms with bandwidth of 3000 MHz](image)
CONCLUSION & FUTURE WORK

Results:

• First reported comparison of experiment and theory for nonlinear SAW in a crystal
• Theory in close quantitative agreement with experiment
• Predictions based on fundamental material properties

Future work:

• Study relationship between nonlinearity matrix elements and waveform distortion [Norfolk ASA meeting]
• Study variation of waveform evolution as function of direction and cut
• Investigate other anisotropic materials
• Investigate piezoelectric effects
The nonlinearity matrix is given by

\[
R_{n_1n_2} = - \sum_{s_1,s_2,s_3=1}^3 \frac{d'_{iklmpq} \beta_i^{(s_1)} \beta_l^{(s_2)} \beta_p^{(s_3)} l_k^{(s_1)} l_m^{(s_2)} l_q^{(s_3)*}}{2[n_1 l_3^{(s_1)} + n_2 l_3^{(s_2)} + (n_1 + n_2) l_3^{(s_3)*}]}
\]

where \( \beta_i^{(s)} = C_s \alpha_i^{(s)} \) and

\[
d'_{iklmpq} = d_{iklmpq} + c_{ikmq} \delta_{lp} + c_{lmkq} \delta_{ip} + c_{pqkm} \delta_{il} .
\]

To compute this expression, the linear problem must first be solved.

Start with linearized wave equation

\[
\frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial x_j} = c_{ijkl} \frac{\partial^2 u_k}{\partial x_j \partial x_l} .
\]  

Next assume SAW solution of form

\[
u_i = \sum_{s=1}^3 C_s \alpha_i^{(s)} e^{ik(l_s \cdot \mathbf{r} - \omega t)}
\]

where \( l_s = \{1, 0, \zeta\} \). Substitute Eq. (2) into Eq. (1) to yield

\[
\rho \frac{c^2 \alpha_i}{\partial t^2} = \tilde{\alpha}_i^{(s)} (c) l_i^{(s)}
\]

Solve Eq. (3) subject to the stress-free surface bound. cond.

\[
\sigma_{i3} \bigg|_{x_3=0} = 0 .
\]

Substituting Eq. (2) into Eq. (4) yields

\[
\frac{i k \tilde{\alpha}_i^{(s)} (c) l_i^{(s)}}{3} \sum_{s=1}^3 C_s \alpha_k^{(s)} (c) l_i^{(s)} = 0 .
\]

These equations can be solved numerically for \( l_i^{(s)}, \alpha_i^{(s)}, \) and \( C_s \).
EVOLUTION OF SPECTRA

Measured spectrum at $x = 5$ mm:

Spectra at $x = 21$ mm:
NONLINEARITY PARAMETERS

Plane wave shock formation distance for sinusoid:

\[ \bar{x} = \frac{1}{|\beta_x| \epsilon_x k} \]

where \( \epsilon_x = \frac{v_{x0}}{c} \) and \( v_x = v_{x0} \sin \omega t \) at \( x = 0 \).

- Calculated shock formation distance:
  \( (v_{x0}=25 \text{ m/s}, \omega/2\pi=50 \text{ MHz}) \)
  \[ \bar{x} = 2.9 \text{ mm} \]

  Meaning:
  Because propagation distance \( \Delta x = 16 \text{ mm} \),
  the pulse is well into the shock formation region.

- Calculated coefficient of nonlinearity:
  \[ \beta_x = -1.0 \]

  Meaning:
  The negative sign indicates that peaks recede and
troughs advance in time, opposite to the distortion
of a sound wave.
WAVEFORMS AT FIRST LOCATION

Measured vertical velocity waveform at $x = 5$ mm:

Calculated longitudinal velocity waveform from linear theory:
EFFECT OF RECONSTRUCTION BANDWIDTH

Consider the vertical velocity waveforms at \( x = 21 \) mm.

Theoretical waveforms with shaded bandwidth of 700 MHz:

\[
\text{Shading prefactor} = \exp\left(-\frac{f}{700}\right)^{16}
\]

Theoretical waveforms with bandwidth of 3000 MHz: